An Errata for *Delay Efficient Sleep Scheduling in Wireless Sensor Networks* [1]

Technical Report

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Matthew J. Miller
Department of Computer Science,
and Coordinated Science Laboratory
University of Illinois at Urbana-Champaign
mjmille2@uiuc.edu

***Abstract*— This document discusses some errors we have found in the NP-complete proof for the Infocom 2005 paper, *Delay Efficient Sleep Scheduling in Wireless Sensor Networks* [1].**

I. Background

This document is intended to supplement the corresponding Infocom paper [1], so we will avoid redundantly including their proof here and instead refer the reader to the original document.

The goal in [1] is to find TDMA slot assignments for sleeping nodes that minimize the maximum end-to-end latency in the network. We will use the notation from their paper. The schedule is cyclic and consists of k slots. Each node is assigned one slot during which it will be awake to receive; it will sleep during the remaining $k-1$ slots unless it is sending data to a neighbor. The slot assignment function is denoted as f. Thus, $f : V \rightarrow [0, \ldots, k-1]$ for graph $G = (V, E)$. If node i wants to send to node j, then it has to wait until the slot which j will be awake to receive. If i and j are assigned the same receive slot, then, when i receives a packet, it must wait an entire cycle before it can send to j (i.e., there can be no more than one packet transmission per slot). Thus, the delay between i and j, $d(i, j)$, is:

$$d(i,j) = \begin{cases} k & \text{, if } f(i) = f(j) \\ (f(j) - f(i)) \mod k & \text{, otherwise} \end{cases} \tag{1}$$

The delay, under slot assignment f, of a path from source S to destination D, $P_f(S, D)$, is simply the sum of the $d(i, j)$ values for each link along the path. The delay diameter of G under slot assignment f, D_f, is defined to be the maximum delay between *any* two nodes in the network.

$$D_f = \max_{i,j \in V} P_f(i, j) \tag{2}$$

Given these definitions, the main problem that the authors address is given in *Definition 2* in their paper [1]:

> ***Definition 2:*Delay Efficient Sleep Scheduling (DESS):** Given a graph $G = (V, E)$ and the number of slots k, find an assignment function $f : V \rightarrow [0 \cdots k-1]$ that minimizes the *delay diameter* i.e.
>
> $$f = \arg\min_{f'} \{D_{f'}\} \tag{3}$$

II. Errors in the Proof

In this section we discuss some errors that we found in the proof from Section IV-A in [1] which result in the proof being incorrect and *not* showing that the problem is NP-complete.[1]

A. Wrong Decision Problem

The first problem, which seems to affect the rest of the proof, is that the decision problem for DESS is stated incorrectly. The statement given in [1] is quoted as follows:

> ***Definition 5:*** $DESS(G, k, f, \Delta)$: Given a graph $G = (V, E)$, number of slots k, a positive number Δ and a slot assignment function $f : V \rightarrow [0, \cdots k-1]$, is $D_f \leq \Delta$.

Notice that there is a critical difference in *Definition 2*, which should be the basis for the decision problem, and *Definition 5*. Namely, in *Definition 2* the goal is to *find* f, whereas in *Definition 5*, f is given and one just needs to verify that $D_f \leq \Delta$. It is obvious that the question asked by *Definition 5* can be answered in polynomial time by running an all-pairs shortest path algorithm and comparing Δ to the path with the largest cost. Thus, *Definition 5* is not NP-complete.

The problem being addressed is:

INSTANCE: A graph $G = (V, E)$ and number of slots k.
SOLUTION: A slot assignment function, $f : V \rightarrow [0, \ldots, k-1]$.
MEASURE: The maximum delay diameter in the network, D_f.

Thus, the corresponding decision problem should have been:

Revised Definition 5: Given an instance of $DESS(G, k, \Delta)$, does a slot assignment function, $f : V \rightarrow [0, \cdots k-1]$, exist such that $D_f \leq \Delta$.

[1] We do not claim that the problem is not NP-complete. It may be. However, the proof given in [1] does not show NP-completeness.

Throughout the remainder of this paper, we will use *Revised Definition 5* as the decision problem that should be used for the proof.

B. Shows "if", but not "only if"

In the proof, they consider a special case of the DESS problem for convenience with $k = 2$ and $\Delta = 4$. They reduce the known NP-complete problem 3-SAT to DESS.[2] Their construction is supposed to show that a 3-SAT formula, F, is satisfiable if and only if a slot assignment function, f, exists in DESS that results in $D_f \leq 4$.

The first part of the "if and only if" statement is true based on their construction: if the instance of the 3-SAT formula is satisfiable, then a slot assignment function, f, *does* exist in DESS that results in $D_f \leq 4$. However, the second part of the statement is not necessarily true: if a slot assignment, f, exists in DESS that results in $D_f \leq 4$, then the corresponding instance of 3-SAT *is not necessarily satisfiable*.

As a simple proof by contradiction, consider Figure 2 from [1]. We introduce slot assignment function f'', which uses the same algorithm in rules 1–3 of their proposed f' function [1], but changes the fourth rule to be:

4) $\forall i \in [1, \cdots m] : f''(X_{i1}) = 0$ and $f''(X_{i2}) = 0$

Using slot assignment function f'', DESS will always have $D_f \leq 4$ regardless of whether or not 3-SAT is satisfiable. Thus, by showing this one contradictory slot assignment function, we have shown that the existence of a slot assignment function that results in $D_f \leq 4$ does not necessarily imply that the corresponding 3-SAT instance is satisfiable.

C. Literal and Compliment in 3-SAT Could Be Assigned the Same Value

Any reduction from 3-SAT must assure that a literal and its compliment cannot be assigned the same value since this is obviously impossible. Unfortunately, their proof places no such restriction on a slot assignment function that results in $D_f \leq 4$. This is demonstrated by the slot assignment function f'' proposed in the previous section. The f'' assignment function will result in $D_f \leq 4$ *and* result in a literal, X, and its compliment, $\overline{X}$, both being set to true, an impossibility.

[2] We ignore that they include a "special case" of f, f', in their proof, since this is based on an incorrect statement of the decision problem as discussed previously.

III. Sketches for a Correct Proof Based on 3-SAT

If the DESS problem is indeed NP-complete, we believe that the authors need to incorporate the following elements in their current construction to achieve a correct proof based on their reduction from 3-SAT:

A. Clause nodes must all have the same slot assignment

In the current construction, nothing enforces that the channel assignment function, f, set all clause nodes to the same slot. Thus, the idea of having a root node, S, is more difficult to use if each clause can be assigned an arbitrary value for a function, f, that results in $D_f \leq \Delta$. Thus, the construction may need to require that if all clause nodes are *not* assigned to the same slot in some function, f, then $D_f > \Delta$.

B. Literal and compliment nodes must have different slot assignments

As shown with our slot assignment function, f'', the current construction does not enforce that a literal node and its complement node must be assigned to different slots if $D_f \leq \Delta$. Thus, the construction must require that a slot assignment function that assigns a literal node and its compliment node to the same slot results in $D_f > \Delta$.

C. Every clause node must connect to at least one variable with a different slot assignment

This is where the essence of the 3-SAT problem is used. The construction must assure that if *every* clause does not connect to at least one variable assigned to a different slot, then $D_f > \Delta$. The current construction seems to attempt to address some aspects of this requirement.

IV. Acknowledgments

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References

[1] G. Lu, N. Sandagopan, B. Krishnamachari, and A. Goel, "Delay Efficient Sleep Scheduling in Wireless Sensor Networks," in *IEEE Infocom 2005*, March 2005.