# An Errata for *Delay Efficient Sleep Scheduling in Wireless Sensor Networks* [1]

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***Abstract*— This document discusses some errors we have found in the NP-complete proof for the Infocom 2005 paper, *Delay Efficient Sleep Scheduling in Wireless Sensor Networks* [1].**

## I. Background

This document is intended to supplement the corresponding Infocom paper [1], so we will avoid redundantly including their proof here and instead refer the reader to the original document.

The goal in [1] is to find TDMA slot assignments for sleeping nodes that minimize the maximum end-to-end latency in the network. We will use the notation from their paper. The schedule is cyclic and consists of $k$ slots. Each node is assigned one slot during which it will be awake to receive; it will sleep during the remaining $k-1$ slots unless it is sending data to a neighbor. The slot assignment function is denoted as $f$. Thus, $f : V \rightarrow [0, \ldots, k-1]$ for graph $G = (V, E)$. If node $i$ wants to send to node $j$, then it has to wait until the slot which $j$ will be awake to receive. If $i$ and $j$ are assigned the same receive slot, then, when $i$ receives a packet, it must wait an entire cycle before it can send to $j$ (i.e., there can be no more than one packet transmission per slot). Thus, the delay between $i$ and $j$, $d(i, j)$, is:

$$d(i,j) = \begin{cases} k & \text{, if } f(i) = f(j) \\ (f(j) - f(i)) \mod k & \text{, otherwise} \end{cases} \quad (1)$$

The delay, under slot assignment $f$, of a path from source $S$ to destination $D$, $P_f(S, D)$, is simply the sum of the $d(i, j)$ values for each link along the path. The delay diameter of $G$ under slot assignment $f$, $D_f$, is defined to be the maximum delay between *any* two nodes in the network.

$$D_f = \max_{i,j \in V} P_f(i, j) \quad (2)$$

Given these definitions, the main problem that the authors address is given in *Definition 2* in their paper [1]:

> ***Definition 2:*Delay Efficient Sleep Scheduling (DESS):** Given a graph $G = (V, E)$ and the number of slots $k$, find an assignment function $f : V \rightarrow [0 \cdots k-1]$ that minimizes the *delay diameter* i.e.

$$f = \arg\min_{f'} \{D_{f'}\} \quad (3)$$

## II. Errors in the Proof

In this section we discuss some errors that we found in the proof from Section IV-A in [1] which result in the proof being incorrect and *not* showing that the problem is NP-complete.[1]

### A. Wrong Decision Problem

The first problem, which seems to affect the rest of the proof, is that the decision problem for DESS is stated incorrectly. The statement given in [1] is quoted as follows:

> ***Definition 5:*** $DESS(G, k, f, \Delta)$: Given a graph $G = (V, E)$, number of slots $k$, a positive number $\Delta$ and a slot assignment function $f : V \rightarrow [0, \cdots k-1]$, is $D_f \leq \Delta$.

Notice that there is a critical difference in *Definition 2*, which should be the basis for the decision problem, and *Definition 5*. Namely, in *Definition 2* the goal is to *find* $f$, whereas in *Definition 5*, $f$ is given and one just needs to verify that $D_f \leq \Delta$. It is obvious that the question asked by *Definition 5* can be answered in polynomial time by running an all-pairs shortest path algorithm and comparing $\Delta$ to the path with the largest cost. Thus, *Definition 5* is not NP-complete.

The problem being addressed is:

**INSTANCE:** A graph $G = (V, E)$ and number of slots $k$.
**SOLUTION:** A slot assignment function, $f : V \rightarrow [0, \ldots, k-1]$.
**MEASURE:** The maximum delay diameter in the network, $D_f$.

Thus, the corresponding decision problem should have been:

***Revised Definition 5:*** Given an instance of $DESS(G, k, \Delta)$, does a slot assignment function, $f : V \rightarrow [0, \cdots k-1]$, exist such that $D_f \leq \Delta$.

[1]We do not claim that the problem is not NP-complete. It may be. However, the proof given in [1] does not show NP-completeness.

Throughout the remainder of this paper, we will use *Revised Definition 5* as the decision problem that should be used for the proof.

### *B. Shows "if", but not "only if"*

In the proof, they consider a special case of the DESS problem for convenience with $k = 2$ and $\Delta = 4$. They reduce the known NP-complete problem 3-SAT to DESS.[2] Their construction is supposed to show that a 3-SAT formula, $F$, is satisfiable if and only if a slot assignment function, $f$, exists in DESS that results in $D_f \leq 4$.

The first part of the "if and only if" statement is true based on their construction: if the instance of the 3-SAT formula is satisfiable, then a slot assignment function, $f$, *does* exist in DESS that results in $D_f \leq 4$. However, the second part of the statement is not necessarily true: if a slot assignment, $f$, exists in DESS that results in $D_f \leq 4$, then the corresponding instance of 3-SAT *is not necessarily satisfiable*.

As a simple proof by contradiction, consider Figure 2 from [1]. We introduce slot assignment function $f''$, which uses the same algorithm in rules 1–3 of their proposed $f'$ function [1], but changes the fourth rule to be:

4) $\forall i \in [1, \cdots m] : f''(X_{i1}) = 0$ and $f''(X_{i2}) = 0$

Using slot assignment function $f''$, DESS will always have $D_f \leq 4$ regardless of whether or not 3-SAT is satisfiable. Thus, by showing this one contradictory slot assignment function, we have shown that the existence of a slot assignment function that results in $D_f \leq 4$ does not necessarily imply that the corresponding 3-SAT instance is satisfiable.

### *C. Literal and Compliment in 3-SAT Could Be Assigned the Same Value*

Any reduction from 3-SAT must assure that a literal and its compliment cannot be assigned the same value since this is obviously impossible. Unfortunately, their proof places no such restriction on a slot assignment function that results in $D_f \leq 4$. This is demonstrated by the slot assignment function $f''$ proposed in the previous section. The $f''$ assignment function will result in $D_f \leq 4$ *and* result in a literal, $X$, and its compliment, $\overline{X}$, both being set to true, an impossibility.

[2] We ignore that they include a "special case" of $f$, $f'$, in their proof, since this is based on an incorrect statement of the decision problem as discussed previously.

## III. Sketches for a Correct Proof Based on 3-SAT

If the DESS problem is indeed NP-complete, we believe that the authors need to incorporate the following elements in their current construction to achieve a correct proof based on their reduction from 3-SAT:

### *A. Clause nodes must all have the same slot assignment*

In the current construction, nothing enforces that the channel assignment function, $f$, set all clause nodes to the same slot. Thus, the idea of having a root node, $S$, is more difficult to use if each clause can be assigned an arbitrary value for a function, $f$, that results in $D_f \leq \Delta$. Thus, the construction may need to require that if all clause nodes are *not* assigned to the same slot in some function, $f$, then $D_f > \Delta$.

### *B. Literal and compliment nodes must have different slot assignments*

As shown with our slot assignment function, $f''$, the current construction does not enforce that a literal node and its complement node must be assigned to different slots if $D_f \leq \Delta$. Thus, the construction must require that a slot assignment function that assigns a literal node and its compliment node to the same slot results in $D_f > \Delta$.

### *C. Every clause node must connect to at least one variable with a different slot assignment*

This is where the essence of the 3-SAT problem is used. The construction must assure that if *every* clause does not connect to at least one variable assigned to a different slot, then $D_f > \Delta$. The current construction seems to attempt to address some aspects of this requirement.

## IV. Acknowledgments

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## References

[1] G. Lu, N. Sandagopan, B. Krishnamachari, and A. Goel, "Delay Efficient Sleep Scheduling in Wireless Sensor Networks," in *IEEE Infocom 2005*, March 2005.